An experimental investigation of the stability of plane shock waves

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The stability of plane shock waves was measured in a shock tube by perturbing the primary shock wave, formed on rupturing the diaphragm, by means of thin wedges. A time-record of the shape of the shock wave after it passed the wedges and travelled along a channel of constant cross-section was obtained by Schlieren photography. Analysis of the photographs enabled the rate at which the shock wave recovered its plane shape to be determined and this, together with the detailed shape of the wave at various instants, was compared with the first-order theory of Freeman (although all the conditions assumed in the theory could not be faithfully reproduced in the experiments).

For shock-wave Mach numbers of 1·165, 1·41 and 1·60, the time-rate of decay of the perturbations was found to agree quite well with the theoretical value, but the amplitudes of the perturbations were much larger than those given by the theory.

The experiments failed to give reliable information about the decay of the perturbations after a large time, owing, it is believed, to flow separation from the sharp corners of the wedges which constituted an additional source of disturbance to the shock waves.

1. Introduction

When a propagating shock wave enters a diverging, converging or parallel channel, it is observed, under certain conditions, that the shock always meets the channel walls perpendicularly and tends to a state of uniform curvature. Thus, on entering a converging or diverging channel the shock tends to cylindrical form, and on entering a parallel channel it tends to plane form.

Perry & Kantrowitz (1951) have noticed that as a shock wave goes from a parallel channel to a converging channel it becomes adjusted to the new conditions very rapidly and passes from plane to cylindrical form in a very short distance. It is also commonly observed that the shock wave produced by bursting the diaphragm in a shock tube attains a plane form after travelling only a few shock-tube diameters, yet it is far from plane when first formed after rupture of the diaphragm. The phenomenon of equalization of curvature along a propagating shock wave has been termed 'shock-wave stability'. The experiments described in this paper were designed to measure the stability of an initially plane shock wave and to compare it with the theoretical predictions of Freeman (1957).

A qualitative explanation of shock-wave stability may be given by considering what occurs when a plane shock propagating along a parallel channel encounters a disturbance on one wall of the channel. For simplicity suppose that the disturbance is a symmetrical wedge (figure 1 *a*). As the shock meets the face *AB*, a cylindrical compression pulse is formed, and the part of the shock between the junction of the pulse and the face *AB* of the wedge is diffracted. This part of the shock is curved and meets *AB* perpendicularly. For this pattern to be formed the wedge angle δ must not exceed a certain value which depends on the strength of the incident shock wave. It will be assumed that δ is always small enough to give rise to the pattern shown in figure 1 *a*. This pattern grows uniformly in time. The

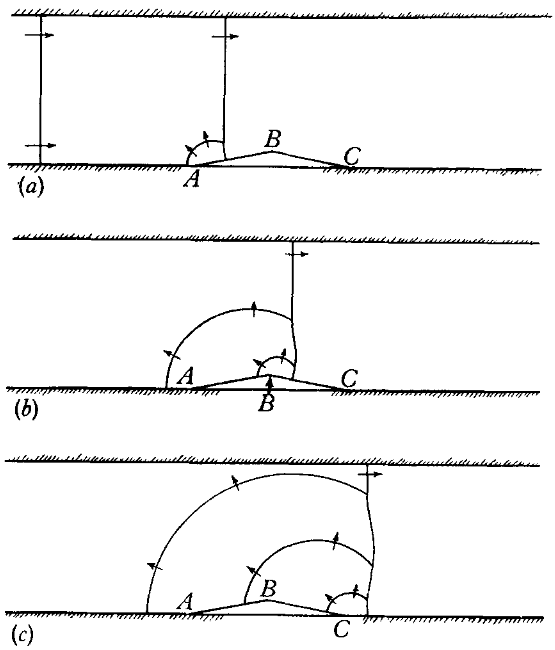

FIGURE 1. Illustration of shock-wave stability.

diffracted portion of the shock eventually reaches the apex *B* of the wedge. The shock is there further diffracted, and an expansion pulse is formed as shown in figure 1 *b*. Finally the shock reaches the corner *C* of the wedge, and another compression pulse is formed, the shock again being diffracted. Each superimposed shock diffraction pattern grows uniformly with time; therefore the curvature at each point of the diffracted part of the shock must decrease with time. The effect of reflexion of the pulses at the channel walls is taken into account by introducing images of the wedge in the walls. Thus, the shape of the shock propagating along the uniform part of the channel is a combination of superimposed diffraction patterns. However, each separate pattern becomes flatter as the shock propagates and the shock becomes plane. Although only a simple disturbance has been considered, the argument is readily extended to the case of a more complex disturbance.

The phenomenon of shock-wave stability has been treated theoretically by Freeman (1957). He considered a plane shock wave propagating along a parallel channel and meeting symmetrical wedges, one on either wall of the channel (figure 2).

The problem is linearized on the assumption of small δ; and, using the results of Lighthill's theory of shock-wave diffraction (1949), the following relation

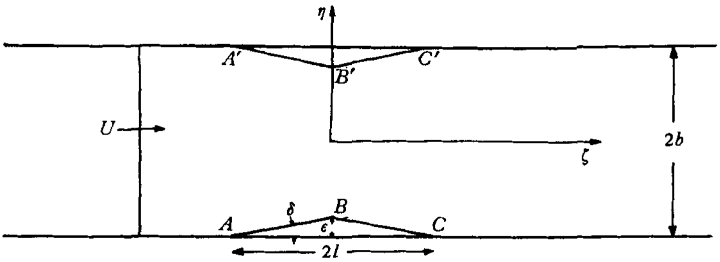

FIGURE 2. The problem treated by Freeman (1957).

between ξ, the perturbation of the shock wave from plane at any point, and the co-ordinates (ζ, η) as in figure 2 is obtained for $\zeta/b \gg 1$:

$$\frac{\xi}{b} = \frac{\epsilon}{l} \frac{\Omega}{(\zeta/b)^{\frac{1}{2}}} \left[F\left\{\frac{\pi}{b}\left(\frac{a_1 k_1}{U}\zeta + \eta\right)\right\} + F\left\{\frac{\pi}{b}\left(\frac{a_1 k_1}{U}\zeta - \eta\right)\right\}\right], \tag{1}$$

where
$$F(\alpha) = 2E(\alpha) - E\left(\alpha + \frac{\pi l}{b}\frac{a_1 k_1}{U}\right) - E\left(\alpha - \frac{\pi l}{b}\frac{a_1 k_1}{U}\right), \tag{2}$$

$$E(\beta) = \sum_{n=1}^{\infty} \frac{(-1)^n \sin n\beta}{n^{\frac{3}{2}}} - \sum_{n=1}^{\infty} \frac{(-1)^n \cos n\beta}{n^{\frac{3}{2}}}, \tag{3}$$

$$k_1 = \left[\frac{6(M_s^2 - 1)}{7M_s^2 - 1}\right]^{\frac{1}{2}}.$$

Here Ω is a function of M_s (the Mach number of the undisturbed shock wave with respect to the velocity of sound ahead of it) and has been given previously by Freeman. Also, a_1 is the velocity of sound behind the shock wave.

The functions F in (1) represent two waves travelling in opposite directions along the shock wave. These waves are due to the disturbances arising from the upper and lower wedges and their respective images.

In connexion with this problem, another effect must be considered. Suppose that the shock travels from the position AA' to BB' (figure 2) in the same time that it takes the junction between the cylindrical compression pulse and the shock wave to travel along the shock a distance equal to the channel width. Then, neglecting the depth of the wedges, when the shock reaches the position BB' the compressions arising at A and A' will just reach B' and B respectively and will coincide with the expansion pulses arising at the apices of the wedges. Similarly, when the shock reaches the position CC' the compression and expansion pulses already formed will coincide with the further compression pulses arising at C and C'. According to the linearized theory, the strength of each compression pulse is proportional to δ and the strength of each expansion pulse is proportional to 2δ. Thus, for $\zeta/b \gg 1$ where the compression and expansion pulses tend to coincide over their whole fronts, the compressions will be annulled by the expansions, and therefore $\xi = 0$.

It can readily be shown that, for this condition of cancellation to hold, $(a_1 k_1/U)(l/b) = 2n$, where n is an integer. In fact, the expression for ξ given by (1) is the leading term of an asymptotic expansion, provided that the above condition of cancellation does not hold. When the above condition does hold, the next highest term in the expansion becomes of greatest importance, in which case the decay of the perturbations would be more rapid than indicated by (1).

On the other hand, if the shock travels from AA' to CC' in the same time as the junction between the first cylindrical compression pulse and the shock wave takes to travel the width of the channel, then the compression pulses arising at A and A' tend to reinforce those arising at C' and C respectively. In this case, the distortions of the shock wave due to the two wedges are superimposed to give a larger distortion. The condition for this reinforcement to occur is $(a_1 k_1/U)(l/b) = 2n+1$.

2. Apparatus and experimental procedure

The phenomenon of shock-wave stability was investigated by allowing plane shock waves produced in a shock tube to travel along a parallel channel with symmetrical wedges on each wall as in figure 2. By taking schlieren photographs of different shock waves, all of the same—or very nearly the same—strength, at various values of ζ, the time-history of the passage of a particular shock through the channel could be deduced.

(i) *The shock tube and instrumentation*

The shock tube used is shown diagrammatically in figure 3. The internal dimensions of the cross-section were 5·875 in. by 1·500 in. Air was used in the tube. The high-pressure section of the tube was left open to the atmosphere and the low-

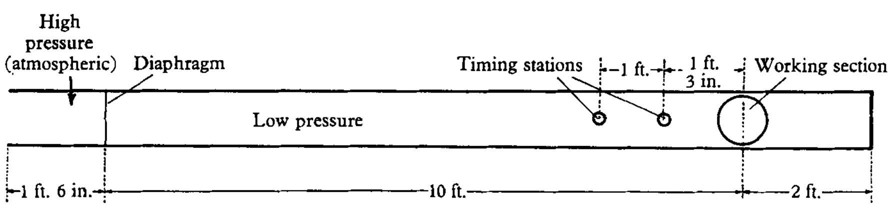

FIGURE 3. Dimensions of shock tube.

pressure section was brought to the desired pressure by a rotary pump. The high- and low-pressure sections were separated by a cellophane diaphragm 0·0015 in. thick. On bursting the diaphragm, a shock wave propagates into the low-pressure section of the tube.

The velocity of the shock wave was found by measuring the time taken by the shock to travel between two stations 1 ft. apart. Each timing station consisted of a small schlieren system with a beam of light across the tube restricted by a narrow vertical slit. The beam, after being focused on a knife-edge, was incident on a photomultiplier. Each small schlieren system was so adjusted that the passage of the shock wave caused light to be deflected off the knife-edge on to the photomultiplier. The resulting signal from the first timing station started an electronic binary counter registering microseconds, and the signal from the second station stopped the counter. The velocity of the shock wave was then readily deduced.

The signal from the second timing station also actuated an electronic delay circuit that could be preset to trigger a spark light source at the working section of the tube at any desired time after the shock had passed the second timing station. The spark gap was the light source for a schlieren system at the working section of the tube; thus, schlieren photographs of shock waves at various positions in the working section could be obtained.

(ii) *The schlieren system at the working section*

The schlieren system is shown diagrammatically in figure 4. The source of light was provided by the discharge across a spark gap of an 0·25 μF condenser charged to 10,000 V. The light was focused on the first knife-edge by the lens L_1 and then

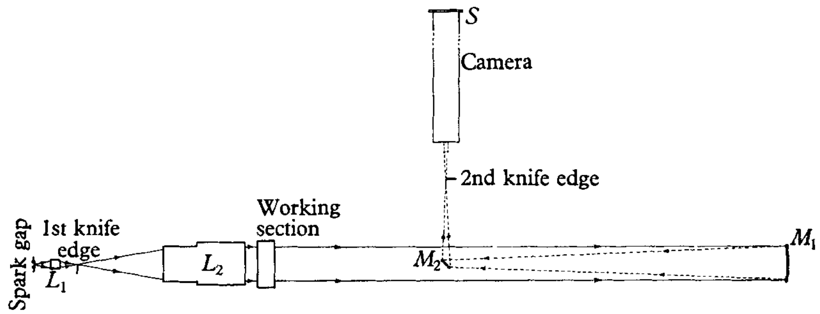

FIGURE 4. Diagram of schlieren system.

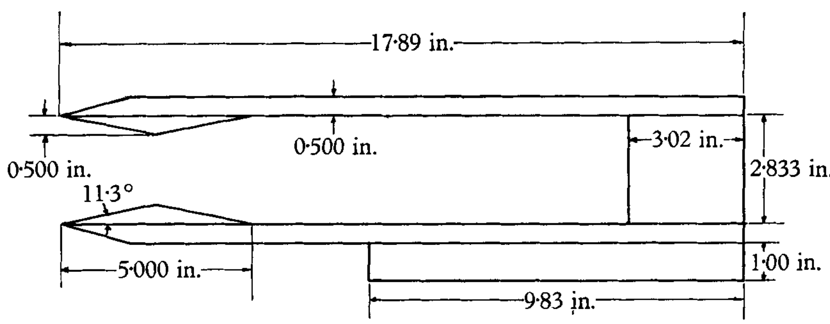

FIGURE 5. Diagram of first wedge system.

collimated by the lens L_2. The collimated beam passed through the working section of the shock tube and was then reflected by the concave mirror M_1 which brought the light to a focus at the second knife-edge. The light then passed into a camera which was focused on the working section of the shock tube.

(iii) *The wedge systems*

One wedge system is shown diagrammatically in figure 5. This was made from ground steel and the whole unit could slide into the shock tube where it could be clamped at any desired position relative to the working section.

After a series of pictures had been obtained with the wedges in one position the whole assembly could be moved to another position and a further series of pictures obtained, thus extending the range of ζ beyond the diameter of the working-section windows, over which information about the behaviour of the shock could be recorded. It was also necessary to have a second and longer wedge system in order to obtain pictures of the flow at distances from the wedges greater than the length of the first wedge system.

The wedges were designed to satisfy as far as possible the conditions assumed in Freeman's linearized theory. In this respect a compromise had to be made: wedges of very small angle, satisfying the conditions of linearized theory, are difficult to make; furthermore, it would be difficult to measure accurately the small distortions in the shock wave caused by wedges of very small angle. The wedges used had an angle of 11·3°, and the dimensions and spacing of the wedges was such that the condition for reinforcement of the disturbances from each wedge held approximately over the range of shock-wave Mach numbers investigated.

(iv) *Measurement of shock-wave shapes*

The co-ordinates of the shock waves were measured directly from the film records by means of a measuring microscope equipped with a travelling stage movable in two perpendicular directions. The positions of the compressions travelling along the shock waves were also noted, except in the cases of shocks a long distance past the wedges when the compressions become too weak to be observable.

It was found by experience that any measurement could be repeated with an accuracy of better than 0·001 in. An estimate of the setting error of the measuring microscope is $\pm$ 0·0005 in., which corresponds to $\pm$ 0·0003 channel widths.

3. Results

Shock waves of Mach numbers 1·165, 1·41 and 1·60 were investigated. A sequence of photographs showing a shock wave of Mach number 1·41 passing over the wedges and along the parallel channel is shown in the composite picture of figure 6, plate 1. The numbers below the picture give the distance in channel widths along the channel measured from the trailing edges of the wedges.

The results obtained by plotting the measured shock-wave co-ordinates are shown in figure 7*a*. In this picture the scale in the direction along the channel has been made 10 times greater than the scale perpendicular to the channel, thus serving to emphasize the shock-wave perturbations. It can be seen from figure 6, plate 1, that the compression waves arising from the leading edge of one wedge and the trailing edge of the opposite wedge are very close together on the shock wave, indicating that the condition for reinforcement holds very nearly. The actual shape of the shock wave at any instant depends on the positions of the compression waves.

Some idea of the rapidity with which the shock tends to plane form may be gained from figure 7*a*. Over the length of the channel shown (approximately 3 channel widths) the compressions travel across the channel $1\frac{1}{2}$ times, and the maximum perturbation of the shock at the end of this short distance is only about one-tenth the perturbation when the shock is just past the wedges.

4. Comparison between theory and experiment

The rather complicated function denoted by F in equation (1) and given explicitly in equations (2) and (3) may be represented adequately by a sine curve provided that $(a_1 k_1/U)(l/b)$ is not too far removed from unity, i.e. if the condition for reinforcement is realized. For the three shock-wave Mach numbers investigated, $(a_1 k_1/U)(l/b)$ was close enough to unity to render the above approximation

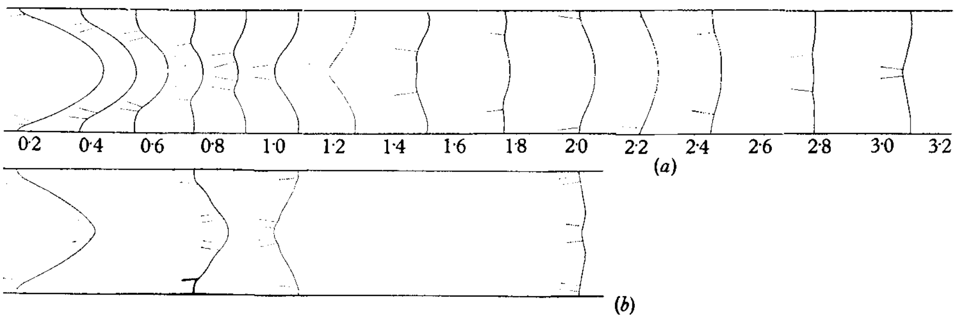

FIGURE 7*a*. Measured shock wave co-ordinates for $M_s = 1{\cdot}41$.
FIGURE 7*b*. Shock wave co-ordinates calculated using Lighthill's theory.
The scale in the direction along the channel is 10 times the perpendicular scale.

M_s	$\frac{a_1 k_1}{U}\frac{l}{b}$	Approximate form for $F(\alpha)$
1·165	0·799	$5{\cdot}11 \sin(\alpha - 3{\cdot}91)$
1·41	0·952	$5{\cdot}64 \sin(\alpha - 3{\cdot}91)$
1·60	0·965	$5{\cdot}64 \sin(\alpha - 3{\cdot}91)$

TABLE 1

for $F(\alpha)$ valid. The values of $(a_1 k_1/U)(l/b)$ and the corresponding values of $F(\alpha)$ for the three shock-wave Mach numbers investigated are shown in table 1. It is seen that, for the shock-wave Mach numbers under consideration, $F(\alpha)$ has the approximate form $A \sin(\alpha - 3{\cdot}91)$. On substituting this approximate value for $F(\alpha)$ in equation (1), the following expression for the perturbation is obtained:

$$\frac{\xi}{b} = \frac{\epsilon}{l}\frac{\Omega}{(\zeta/b)^{\frac{3}{2}}}\, 2A \sin\left(\frac{\pi a_1 k_1}{U}\frac{\zeta}{b} - 3{\cdot}91\right)\cos\left(\frac{\pi\eta}{b}\right). \tag{4}$$

According to this expression, the perturbation is a standing wave in the form of a cosine curve.

Direct comparison between the theoretical expression (4) and the experimental results cannot be made because the position of the plane unperturbed shock from which the perturbations must be measured is not known accurately enough. In order to overcome this difficulty, a quantity called the 'total perturbation' has been introduced. This is defined as the difference between the extreme positions of a perturbed shock wave measured in the direction of propagation (figure 8).

The theoretical expression for the magnitude of the total perturbation ψ is

$$\psi = A\,\frac{0{\cdot}1414\Omega}{(x+0{\cdot}883)^{\frac{3}{2}}}\left|\sin\left[\frac{\pi a_1 k_1}{U}(x+0{\cdot}883) - 3{\cdot}91\right]\right|,$$

where all lengths have been made non-dimensional by dividing by the channel width $2b$. In this system, x is the distance along the channel measured from the trailing edges of the wedges. Experimentally, the total perturbation is readily determined from the plotted shock shapes.

4.1. *Analysis of the experimental results*

For comparison with theory, a function was chosen of a form similar to the theoretical expression, and the parameters in this function were evaluated to give the best fit to the experimental results by the method of least squares.

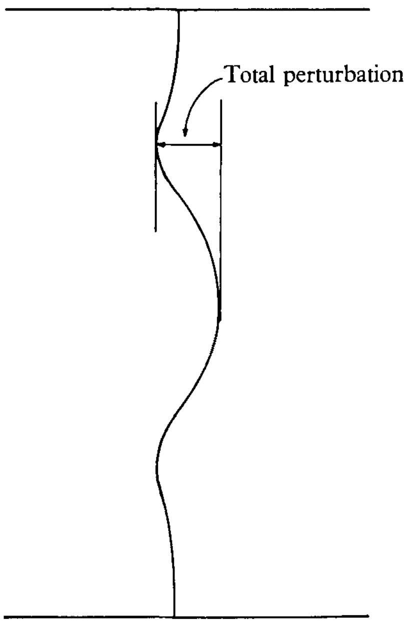

FIGURE 8. Sketch defining total perturbation of shock wave.

The function chosen to fit the experimental results was

$$\psi = \frac{G}{(x+0\cdot883)^n}\,|\sin[m(x+0\cdot883)+p]|,$$

where n, G, m and p are the parameters to be evaluated. m and p are easily determined; on the other hand, to evaluate G and n it was found convenient to minimize $\Sigma(\log\psi - \log\psi')^2$, where ψ' is the measured total perturbation. Evidently, the values of G and n obtained in this way would not be exactly the same as those found by minimizing $\Sigma(\psi-\psi')^2$, but they led to curves which fitted the experimental points quite well.

This analysis was carried out for shock waves of Mach numbers 1·41 and 1·60, but proved unsuccessful for shock waves of Mach number 1·165 (see figures 9–11). From the experimental results for $M_s = 1\cdot165$ (figure 11), it can be seen that the period of oscillation of total perturbation did not remain constant; therefore it was impossible to find constant values for m and p, and the analysis failed.

The results for $M_s = 1{\cdot}41$ and $1{\cdot}60$ in figures 9 and 10 show that for values of x greater than about 4 the measured perturbations behaved in an erratic manner and ceased to decrease with increasing x. Possible reasons for this behaviour will

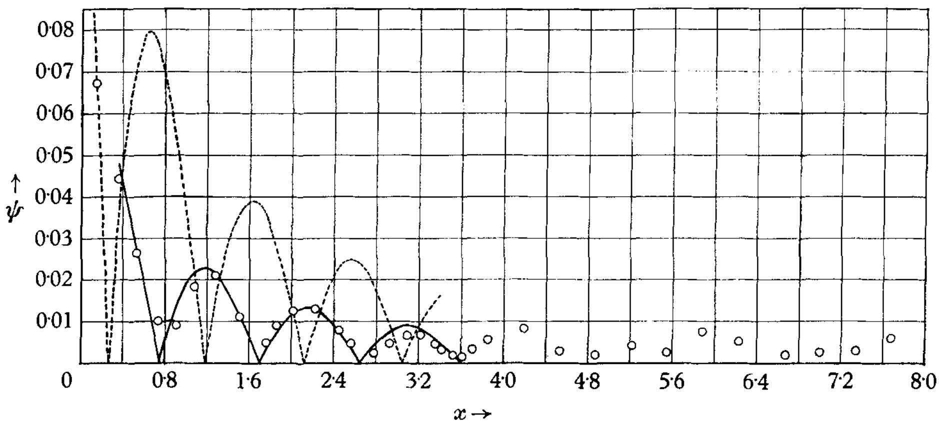

FIGURE 9. Graph of total perturbation *vs* distance. ○, Experimental results; —, fitted curve; ---, theoretical curve.

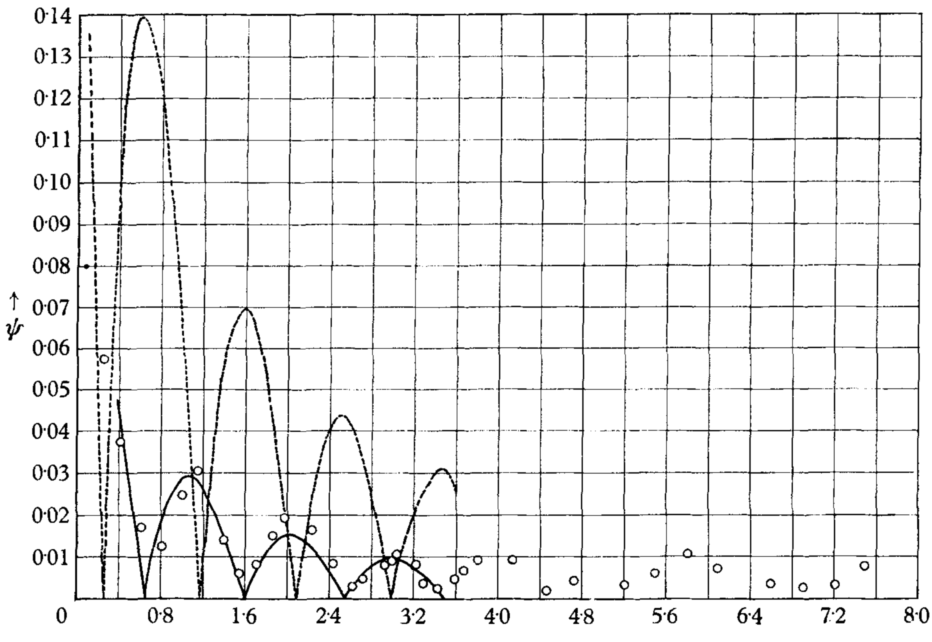

FIGURE 10. Graph of total perturbation *vs* distance. ○, Experimental results; —, fitted curve; ---, theoretical curve.

be discussed later. These anomalous results were not used in the analyses, and the fitted curves are shown extending over the results that were used. The perturbations of the shock waves before the compressions from the trailing edges of the wedges had travelled across the channel once were not included in the analyses either for, at such an early stage in the stabilization of the shock wave, the regions of the shock near the junctions of the compressions and the shock had not expanded far enough to affect much of the shock wave.

The results found for n and G at $M_s = 1{\cdot}41$ and $1{\cdot}60$ are tabulated in table 2 together with the theoretical values of these quantities given by Freeman's theory.

In assessing how well the 'best' curves fit the experimental results, the errors in the determination of the total perturbations must be considered. There are two sources of error: (*a*) Each point on the graph is determined from measurements

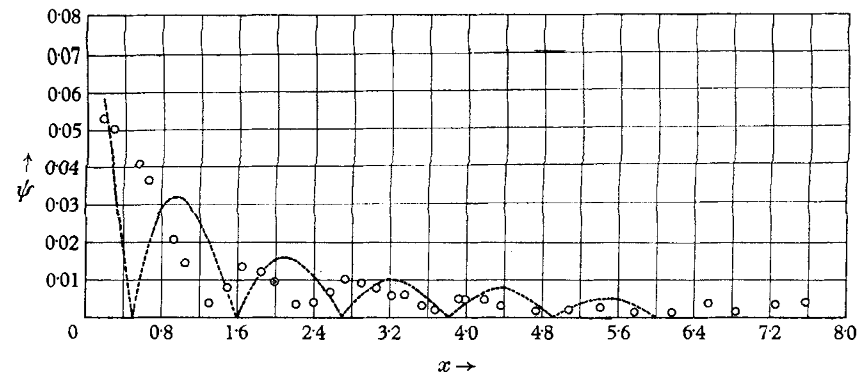

FIGURE 11. Graph of total perturbation *vs* distance. ○, Experimental results; —, fitted curve; – – –, theoretical curve.

	n		G	
M_s	Experiment	Theory	Experiment	Theory
1·41	1·45	1·5	0·066	0·157
1·60	1·65	1·5	0·090	0·273

TABLE 2

of different shock waves; the Mach numbers of these shock waves all agree to better than 1 %. The effect of the small scatter in shock-wave Mach numbers was estimated by obtaining pictures of shock waves of the same nominal Mach number at the same position in the channel. This was done for $M_s = 1{\cdot}60$, and the effect on the perturbation of the scatter in Mach number was found to be very small indeed, much less in fact than the deviation of some of the experimentally determined points from the fitted curve. (*b*) The error in the actual measurement of the perturbation has already been discussed. This is less than the deviation of some of the points from the fitted curve.

It seems likely that the form of the function chosen to fit the experimental results is too simple to give an adequate description of the phenomenon. It should be borne in mind that the form of the function chosen to fit the experimental results is the same as the form of the theoretically derived function which is valid only at a large number of channel widths beyond the wedges. Although the

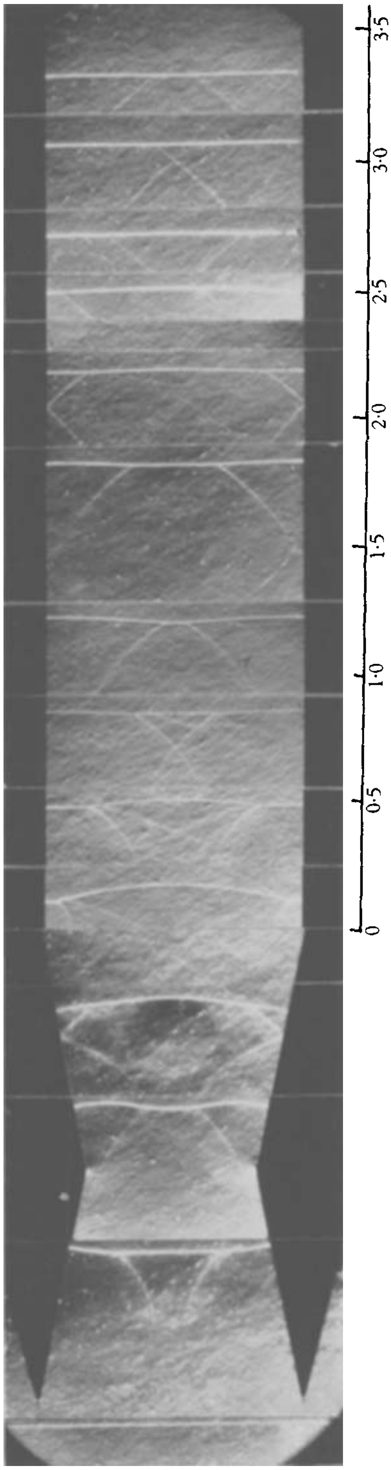

FIGURE 6 (plate 1). Photographs of shock wave at $M_s = 1{\cdot}41$.

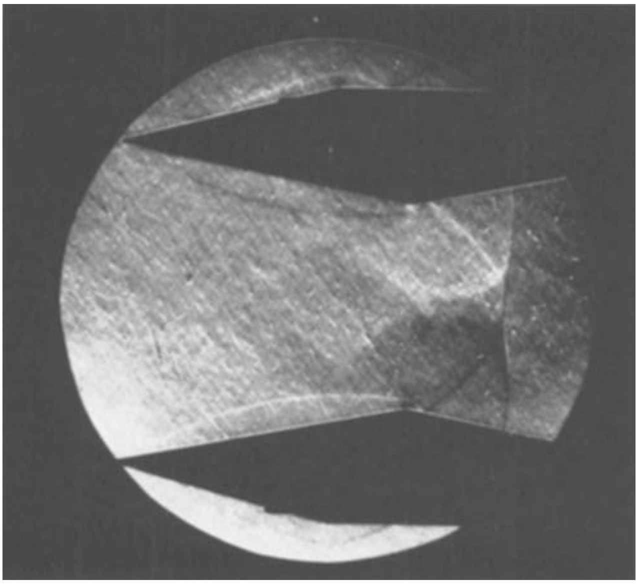

FIGURE 12 (plate 2). Photograph showing diffusion of expansion wave.

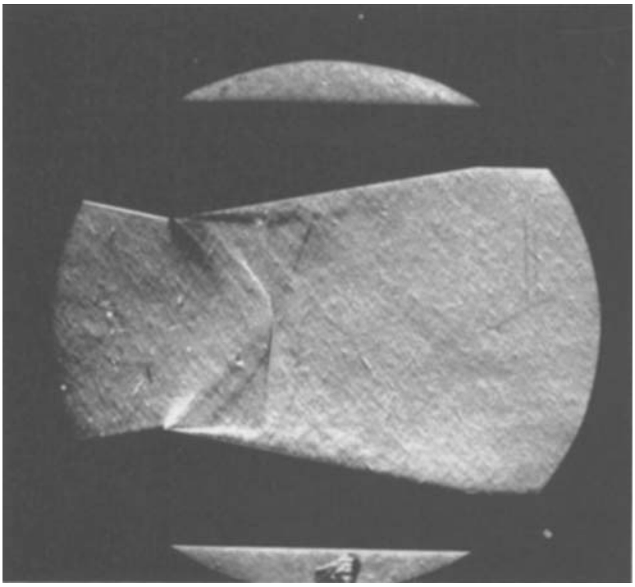

FIGURE 13 (plate 2). Photograph showing wake behind shock wave.

theory is valid only at large distances beyond the wedges, there is agreement with experiment in certain respects. The points on which theory and experiment agree are summarized below:

(*a*) The period of oscillation of the theoretical and experimental values of ψ compare well.

(*b*) The experimentally determined law for the decay of the perturbation with distance compares well with the law given by theory, viz. $\xi \propto \zeta^{-\frac{1}{2}}$.

The points of difference between experiment and theory are as follows:

(*c*) The theoretical expression for the perturbation given by equation (4) shows that the shock shape should be a cosine curve for the shock-wave Mach numbers under consideration. This was not observed experimentally. The theory also indicates that the shock perturbations should become small, i.e. $O(\zeta/b)^{-\frac{3}{2}}$, at regular intervals along the channel; observation showed that, although the shock did not become plane, the perturbations did become very small at regular intervals.

(*d*) There is a phase difference between the theoretical and experimental results.

(*e*) The maximum total perturbations predicted by theory are considerably greater than those observed experimentally.

Comparison between the observed shock shapes and those given by direct application of Lighthill's theory of diffraction of blast

Freeman's theory is based on Lighthill's theory but holds good only for $x/b \gg 1$. Direct application of Lighthill's theory may be expected to give better agreement with experiment near the origin. Shock shapes were calculated for $M_s = 1{\cdot}41$ using Lighthill's theory: these calculated shock shapes are shown in figure 7*b*, where they can readily be compared with the experimental shapes of figure 7*a*. (The dotted lines are expansion pulses which arise from the apex of each wedge.)

From figure 7*b* it is seen that the curvatures of the theoretically determined shock shapes tend to be concentrated in smaller regions of the shock waves than observed in the experiments. This is due to the fact that the theory, which is linear, treats the expansion waves as localized pulses travelling along the shock wave whereas, as can be seen from figure 12, plate 2, the expansion wave tends to become diffuse and meets the shock over a region of its length rather than at one point. This spreading out of the expansion waves (which is explained, at least qualitatively, by the non-linear theory of Whitham 1957) tends to make the curves in the shock shape smoother than those given by the linearized theory.

4.2. *Discussion on the irregular character of the perturbations at the largest values of x investigated*

From figures 9–11 it can be seen that the perturbations cease to decay in a regular manner beyond a certain distance along the channel. It is here suggested that this erratic behaviour may be attributed to the formation of a wake at the apex of each wedge after the passage of the shock wave. These wakes will lead to attenuation of the shock wave since they represent sinks of energy. It has been shown theoretically by Hollyer (1956), and by Trimpi & Cohen (1955), that the growth of a boundary layer on the shock tube walls can account for observed shock-wave attenuation. The theoretical work treats the problem one-dimensionally and serves to give an overall attenuation of the shock wave, whereas in the present discussion interest centres on the two-dimensional effects of the wakes on the shock wave. A picture of the separated flow over the wedges is shown in figure 13, plate 2. This picture shows the situation 685 μsec after a shock wave of Mach number 1·41 had passed the apexes of the wedges. The well-developed wake can be observed and it is to be expected that this will affect the shock wave propagating along the channel. A certain time must elapse after the passage of the shock wave before a wake is formed. Whilst the wake is forming and also when it has formed, the main shock wave will be attenuated by propagation of expansion waves from the wake. This process will take a certain length of time after the shock wave has passed the wedges; therefore it may be expected that any effect the wake would have on the shock would only become apparent after the shock wave had travelled some distance along the channel. It is in fact observed that the results become erratic at a certain distance past the wedges.

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REFERENCES

FREEMAN, N. C. 1957 On the stability of plane shock waves. *J. Fluid Mech.* **2**, 397–411.

HOLLYER, R. N. JR. 1956 Attenuation in the shock tube. I. Laminar flow. *J. Appl. Phys.* **27**, 254–61.

LIGHTHILL, M. J. 1949 The diffraction of blast. I. *Proc. Roy. Soc.* A, **198**, 454–70.

PERRY, R. W. & KANTROWITZ, A. 1951 The production and stability of converging shock waves. *J. Appl. Phys.* **22**, 878–86.

TRIMPI, R. L. & COHEN, N. B. 1955 A theory for predicting the flow of real gases in shock tubes with experimental verification. *Tech. Note Nat. Adv. Comm. Aero., Wash.*, no. 3375.

WHITHAM, G. B. 1957 A new approach to problems of shock dynamics. Part I. Two-dimensional problems. *J. Fluid Mech.* **2**, 145–71.